

Central Dominance and the Confinement Mechanism in Gluodynamics

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Abstract

New topological objects, which we call center monopoles, naturally arise in the Maximal Center Projection of SU(3) gluodynamics. The condensate of the center monopoles is the order parameter of the theory.

1 Introduction

The nonlinear structure of strong interaction physics explains why analytical approaches to the full investigation of the QCD internal structure are absent. Therefore QCD must be investigated by a combination of computer simulations and semi-analytical approaches. Today our knowledge of gluodynamics includes several beautiful structures arising in the QCD vacuum. Among these structures we mention instantons, Maximal Abelian monopoles, center vortices, and gluonic strings. These objects influence the main physical effects such as asymptotic freedom, the creation of glueballs and confinement.

In this work we investigate one of the structures mentioned above, corresponding to the center vortices in $SU(3)$ gluodynamics (the $SU(2)$ case was investigated in [1]). This phenomenon was discovered by Greensite et al. (see, for example, [2]) and can be seen through the so-called Maximal Center Projection. Within this picture it was shown that string-like center vortices interact with quarks via topological Aharonov - Bohm forces. Moreover, these forces are responsible for confinement [4].

It is already a tradition to describe confinement by the dual superconductor mechanism, which was proposed by Mandelstam. It seems that this mechanism is clearly realized in the $SU(2)$ simplification of QCD using the so-called Maximal Abelian projection [5]. After this projection monopoles appear, which are condensed in the confinement phase. Due to this condensation the color force lines are constricted into a string, which connects a quark and an antiquark. This string has nonzero tension, which leads to the confinement of quarks. A great number of physicists hope that the Maximal Abelian projection works in the same way in the real $SU(3)$ theory. We do not know if they are right or not, but in $SU(3)$ gluodynamics after the Maximal Abelian projection not one, but two monopoles appear, and even if the dual superconductor mechanism works here, it must be complicated [3].

According to the Center Dominance hypothesis center vortices are responsible for confinement. Thus the following question arises in a natural way during the investigation of the Maximal Center projection: What is the connection between center vortices and the dual superconductor picture. To answer this question we construct monopole-like objects from center vortices. We call them center monopoles. The condensate of center monopoles is the order parameter. It is different from zero in the confinement phase of the $SU(3)$ theory. Thus one may expect that the center monopole is the

monopole which plays a role in the dual Meissner mechanism.

2 The Maximal Center Projection.

We consider $SU(3)$ gluodynamics with the Wilson action $S(U) = \beta \sum_{\text{plaq}} (1 - 1/3 \text{Re Tr } U_{\text{plaq}})$. Here the sum is over the plaquettes of the lattice. If the given plaquette consists of the links $[xy], [yz], [zw], [wx]$ then $U_{\text{plaq}} = U_{[xy]} U_{[yz]} U_{[zw]} U_{[wx]}$.

The Maximal Center Projection makes the link matrix U as close as possible to the elements of the center Z_3 of $SU(3)$: $Z_3 = \{\text{diag}(e^{(2\pi i/3)N}, e^{(2\pi i/3)N}, e^{(2\pi i/3)N})\}$, where $N \in \{1, 0, -1\}$. In this work we use the so-called indirect version of the Maximal Center Projection. This procedure works as follows.

First, make the functional

$$Q_1 = \sum_{\text{links}} (|U_{11}| + |U_{22}| + |U_{33}|) \quad (1)$$

maximal with respect to the gauge transformations $U_{xy} \rightarrow g_x^{-1} U_{xy} g_y$, thus fixing the Maximal Abelian gauge. As a consequence every link matrix becomes almost diagonal.

Secondly, to make this matrix as close as possible to the center of $SU(3)$, make the phases of the diagonal elements of this matrix maximally close to each other. This is done by minimizing the functional

$$Q_2 = \sum_{\text{links}} [(1 - \cos(\text{Arg}(U_{11}) - \text{Arg}(U_{22}))) + (1 - \cos(\text{Arg}(U_{11}) - \text{Arg}(U_{33}))) + (1 - \cos(\text{Arg}(U_{22}) - \text{Arg}(U_{33})))]. \quad (2)$$

with respect to the gauge transformations. This gauge condition is invariant under the central subgroup Z_3 of $SU(3)$.

The center vortices are defined as follows. After fixing the Maximal Center gauge we define the integer-valued link variable N :

$$\begin{aligned} N_{xy} &= 0 & \text{if } (\text{Arg}(U_{11}) + \text{Arg}(U_{22}) + \text{Arg}(U_{33}))/3 \in] - \pi/3, \pi/3], \\ N_{xy} &= 1 & \text{if } (\text{Arg}(U_{11}) + \text{Arg}(U_{22}) + \text{Arg}(U_{33}))/3 \in]\pi/3, \pi], \\ N_{xy} &= -1 & \text{if } (\text{Arg}(U_{11}) + \text{Arg}(U_{22}) + \text{Arg}(U_{33}))/3 \in] - \pi, -\pi/3]. \end{aligned} \quad (3)$$

In other words $N = 0$ if U is close to 1, $N = 1$ if U is close to $e^{2\pi i/3}$ and $N = -1$ if U is close to $e^{-2\pi i/3}$.

Next we define the plaquette variable:

$$\sigma_{xyz} = N_{xy} + N_{yz} - N_{zx} - N_{xz} \quad (4)$$

We introduce the dual lattice and define the variable σ^* dual to σ : if plaquette $^*\Omega$ is dual to plaquette Ω , then $\sigma_{^*\Omega}^* = \sigma_\Omega$. One can easily check that the variable σ represents a closed surface. This surface is known as the worldsheet of the center vortex.

We express the $SU(3)$ gauge field U as the product of $\exp((2\pi i/3)N)$ and V , where V is the $SU(3)/Z_3$ variable $(\text{Arg}(V_{11}) + \text{Arg}(V_{22}) + \text{Arg}(V_{33}))/3 \in] - \pi/3, \pi/3]$. Then $U = \exp((2\pi i/3)N)V$

After that we represent the action of the Wilson loop C as follows:

$$W_C = \Pi_C U = \exp((2\pi i/3)L(C, \sigma))\Pi_C V \quad (5)$$

The term $(2\pi i/3)L(C, \sigma)$ is known as the Aharonov - Bohm interaction term. The quantity $L(C, \sigma)$ is the linking number of the loop C and the closed surface σ^* .

The content of the center dominance hypothesis is that after the Maximal Center projection the Aharonov - Bohm interaction term by itself causes confinement and produces the full string tension.

The center monopole is the Z_3 analogue of the monopole in $U(1)$ theory. Let us recall that monopoles in $U(1)$ theory are constructed as loops on which the force lines of the gauge field end. It is well known that in electrodynamics the Maxwell equations $dF = 0$ restrict the existence of magnetic charges. But in the compact theory values of F which differ from each other by 2π , are equivalent. Thus the correct field strength is $F \bmod 2\pi$ and $^*d(F \bmod 2\pi) = 2\pi j_m$, where j_m is the monopole current.

The Aharonov - Bohm interaction between the center vortex and the quark depends only on $[\sigma] \bmod 3$. Here σ is the Z_3 analogue of the $U(1)$ field strength. The variable $[\sigma] \bmod 3$ represents the surface with boundary. This boundary is a closed line. We assume that this line represents the world trajectory of the particle, which we call a center monopole:

$$3j_m = ^*d([\sigma] \bmod 3) = \delta([\sigma^*] \bmod 3). \quad (6)$$

(Here we use the notations of differential forms on the lattice. For the definition of these notations see, for example, [6].)

We suggest the reader to consider the center monopole as the monopole which condensation leads to formation of the quark-antiquark string according to the dual superconductor mechanism. The results of the next section partially justify this hypothesis. In particular, we find that in the finite temperature theory the condensate of the center monopoles is the order parameter. The center monopoles are condensed indeed in the confinement phase.

3 Numerical results

We used a lattice of size $16^3 \times 4$. The confinement - deconfinement phase transition for this lattice is at $\beta = 5.69$ approximately [7]. Our results are as follows

1. The center vortices are condensed in the confinement phase, and not condensed at high temperature. This follows from the consideration of the probability that two points are connected by the string worldsheet. We find for this probability: $\rho(x, y)_{\text{vort}} \rightarrow C_{\text{vort}}(\beta)$ at $|x - y| \rightarrow \infty$. We observe, that C_{vort} is equal to 1 in the confinement phase, and falls to zero in the deconfinement phase. (The solid line in Fig. 1.)
2. The density of the center vortices is represented in Fig. 2.
3. The fractal dimension of center vortices which is given by $D = 1 + 2A/L$, where A is the number of plaquettes and L is the number of links on the string, is represented in Fig. 3.
4. The center monopoles are condensed in the confinement phase, but not condensed in the deconfinement phase. This follows from a consideration of the probability that two points are connected by a monopole worldline. We find for this quantity: $\rho(x, y)_{\text{mon}} \rightarrow C_{\text{mon}}(\beta)$ at $|x - y| \rightarrow \infty$. We observe, that C_{mon} is equal to 0 in the deconfinement phase, and different from 0 in the confinement phase. The condensate as a function of β is represented in Fig. 1 by the dashed line.
5. In addition we notice here, that according to the percolation properties, the center vortices and center monopoles are distributed homogeneously. This follows from the fact that the probability of two points

to be connected by the worldsheet or the worldline of these objects is constant for all distances for center monopoles and center vortices.

4 Conclusions

The Maximal Abelian Projection which has been used actively lately in the investigation of strong interaction physics, leads to the existence of two monopoles within $SU(3)$ gauge theory. Thus the dual superconductor mechanism of confinement becomes complex and unnatural. In our work we use the Maximal Center Projection and find that after applying this procedure a new interesting object arises. We call it the Center monopole. It turns out that this monopole is condensed in the confinement phase and is not condensed in the deconfinement phase. Thus we believe this object to be a good candidate for the monopole that works in the dual superconductor mechanism.

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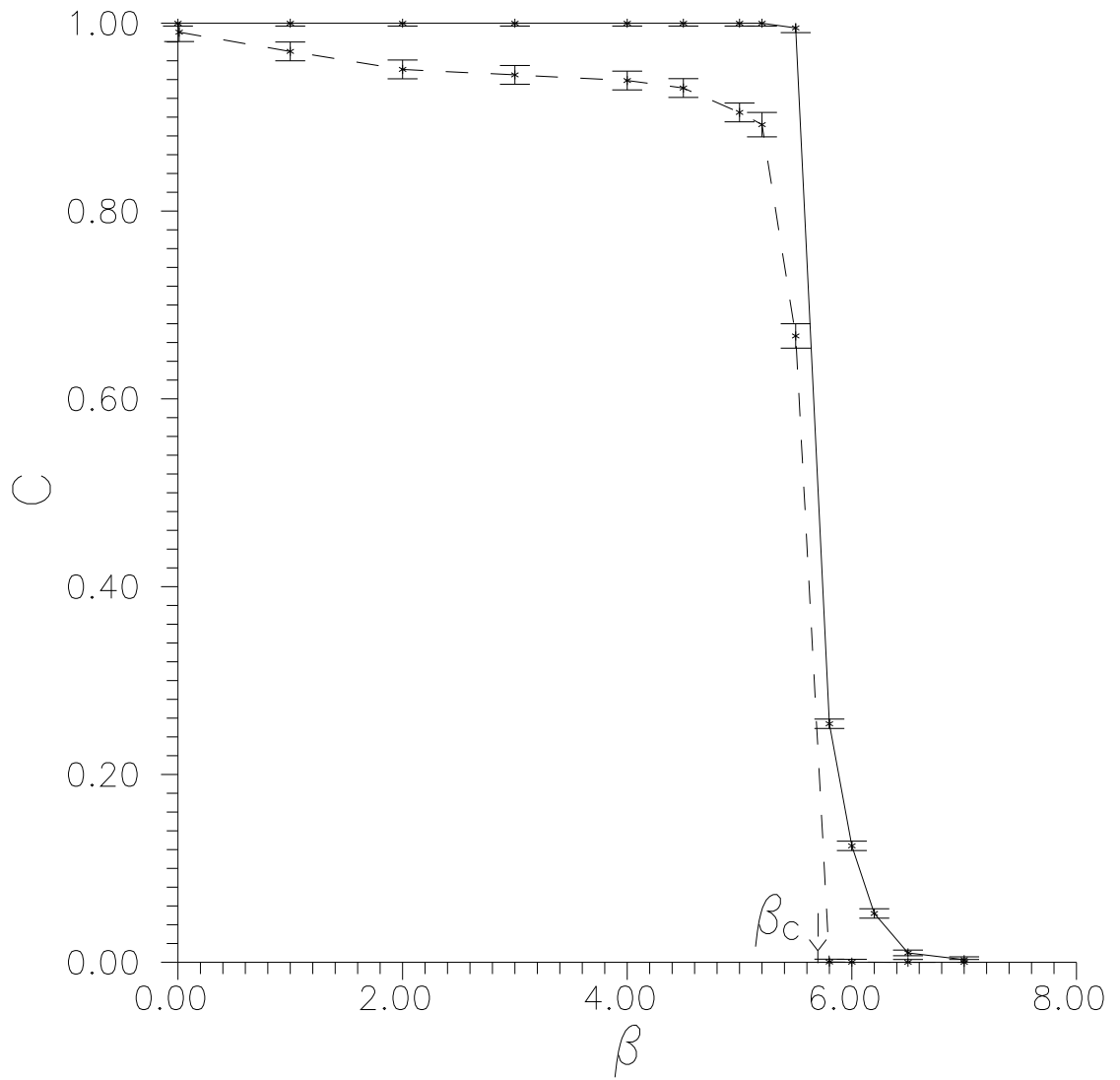


Figure 1: Percolation properties of center vortices (solid curve) and center monopoles (dashed curve).

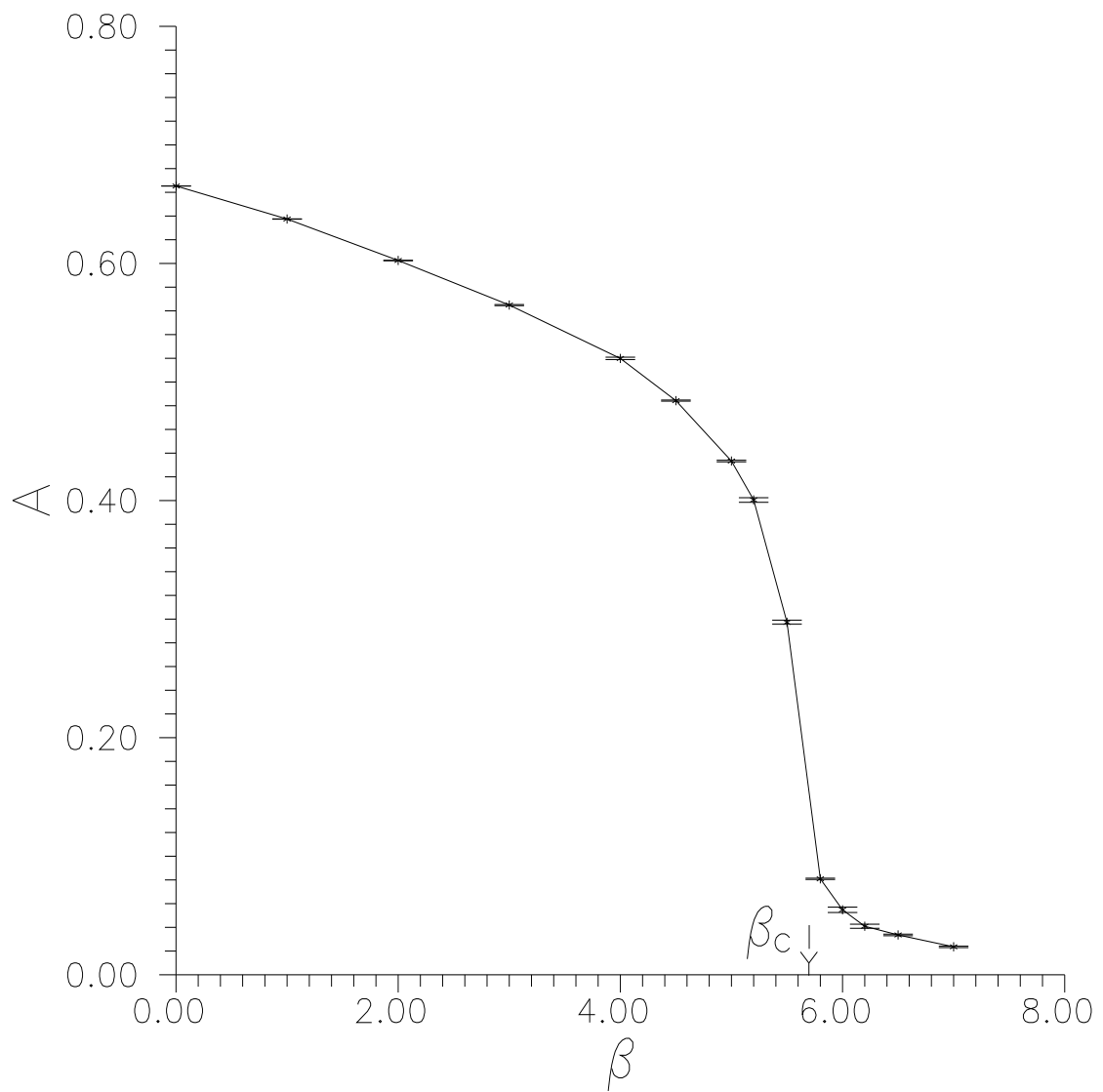


Figure 2: The density of the center vortices.

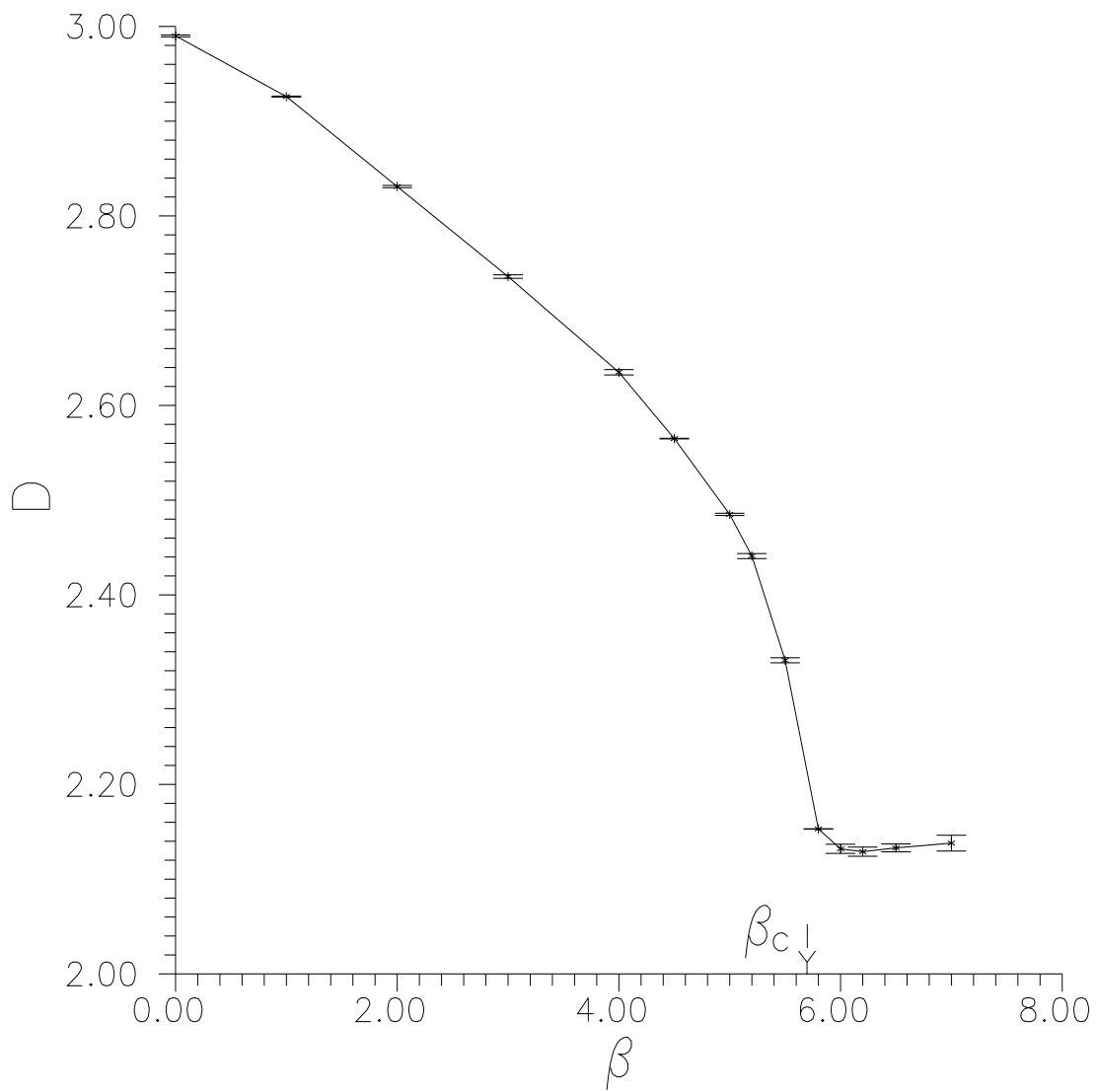


Figure 3: The fractal dimension of the center vortices.